UNIT-III III-unit

TURNING MOMENTS DIAGRAMS:

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Flywheel:- "A flywheel acts as a energy reservoir for storing and releasing mechanical energy of the engine".

During the working stroke in the engine, when the energy produced is more than the requirement the speed of the engine increases. This energy produced during working stroke is distributed among the idle strokes which results in the decrease of engine speed. That is not desirable. To overcome this fluctuation in speed, the function of flywheel comes into picture.

During the power or working stroke the flywheel absorbs the excess energy and during the idle stroke it gives away the same to the engine as per the load requirement.

Co-efficient of fluctuation of speed:- "It is defined as the ratio of maximum fluctuation of speed to the mean speed". It is denoted by C_s

$$\therefore C_s = \frac{N_1 - N_2}{N} \qquad \left[\because N = \frac{N_1+N_2}{2}\right]$$

$$= \frac{N_1 - N_2}{\frac{N_1+N_2}{2}} \Longrightarrow \frac{2[N_1 - N_2]}{N_1+N_2}$$

$$\because \omega_1 = \frac{2\pi N_1}{60} \qquad N_1 = \frac{\omega_1 \times 60}{2\pi} \qquad N_2 = \frac{\omega_2 \times 60}{2\pi}$$

$$= \frac{2\left[\frac{\omega_1 \times 60}{2\pi} - \frac{\omega_2 \times 60}{2\pi}\right]}{\frac{\omega_1 \times 60}{2\pi} + \frac{\omega_2 \times 60}{2\pi}}$$

$$= \frac{\frac{2\times \cancel{60}}{\cancel{2\pi}}[\omega_1 - \omega_2]}{\frac{\cancel{60}}{\cancel{2\pi}}[\omega_1 + \omega_2]} \Longrightarrow \boxed{C_s = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}}$$

where

N_1 = Maximum speed ; N_2 = Minimum speed

N = mean speed $= \frac{N_1+N_2}{2}$

ω_1 = Maximum Angular velocity

ω_2 = minimum Angular Velocity

Co-efficient of fluctuation of energy:- "It may be defined as the ratio of maximum fluctuation of energy to work done per cycle". It is denoted by C_E

$$\therefore C_E = \frac{\text{Maximum fluctuation of energy}}{\text{work done per cycle}}$$

Some Relations:-

1). work done per cycle $= \frac{P \times 60}{n}$

2). moment of Inertia of a solid flywheel disc $I = \frac{mD^2}{8}$

3). Mass of flywheel rim $m = A \cdot \pi D \cdot \rho$

4). Hoop stress $\sigma = \rho V^2 \Rightarrow \rho \left[\frac{\pi DN}{60}\right]^2$

$A = b \times t$

where

P = power transmitted ...... watts

n = Number of working strokes perminute

$= \frac{N}{2}$ in case of four stroke I-C engine

A = cross sectional area of flywheel rim.

b = width of rim

t = thickness of rim.

σ = hoop stress N/m^2 due to centrifugal force

ρ = density of rim material Kg/m^3

D = Mean diameter of rim, ... m

V = linear velocity of flywheel .... m/sec

N = speed of flywheel in r.p.m.

$$P = \frac{2\pi NT}{60} \quad ; \quad \omega = \frac{2\pi N}{60}$$

$$V = \frac{\pi DN}{60}$$

where P = power .... watt

ω = Angular velocity .... rad/sec

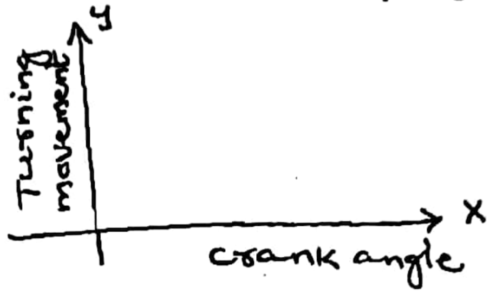
V = Linear velocity .... m/s

T = Torque .... N-m

Turning movement diagram. mention its uses?

Turning movement diagram is a diagram which gives relationship between the turning movement on crank for various position of the crank. It is also called T-m diagrams & M-θ diagrams. This diagram is obtained by plotting turning movement on Y-axis and crank angle on X-axis. This diagram is different for different engines. The variation of turning movement w.r.t crank angle can be determined with the help of these diagrams.

Uses:-



(i) Area of turning movement diagram per cycle gives the work done per cycle. when this is multiplied by number of cycles per minute power developed by the engine is obtained.

(ii) The maximum height of the turning movement diagram gives the maximum torque acting on the crank shaft.

(iii) when area of turning movement diagram is divided by length of the base, mean torque & mean turning movement is obtained.

Turning moment diagrams Refer page NO: 8 →

$\boxed{\text{Exam}}$

Derive the equation $K = \frac{e}{2E}$ where K = co-efficient of fluctuation of speed, explain maximum fluctuation of energy and kinetic energy :-

Let

m = Mass of the flywheel in kg

K = Radius of gyration of flywheel in m

N_1 = Maximum speed

N_2 = Minimum speed

N = Mean speed $= \frac{N_1+N_2}{2}$ & $\omega = \frac{\omega_1+\omega_2}{2}$

$$= \frac{1}{2} I\omega_1^2 - \frac{1}{2} I\omega_2^2$$

$$= \frac{1}{2} I\left[\omega_1^2 - \omega_2^2\right] \qquad \left[\because a^2-b^2 = (a+b)(a-b)\right]$$

$$= \frac{1}{2} I(\omega_1 + \omega_2)\left[\omega_1 - \omega_2\right]$$

$$= I\left[\frac{\omega_1+\omega_2}{2}\right]\left[\omega_1 - \omega_2\right] \qquad \left[\because \frac{\omega_1+\omega_2}{2} = \omega\right]$$

$$= I\cdot\omega\cdot\left[\omega_1 - \omega_2\right]$$

Multiplying and dividing both sides by 'ω', we get

$$\Delta E = I\omega\left[\omega_1 - \omega_2\right]\frac{\omega}{\omega} \Longrightarrow I\omega^2\left[\frac{\omega_1-\omega_2}{\omega}\right]$$

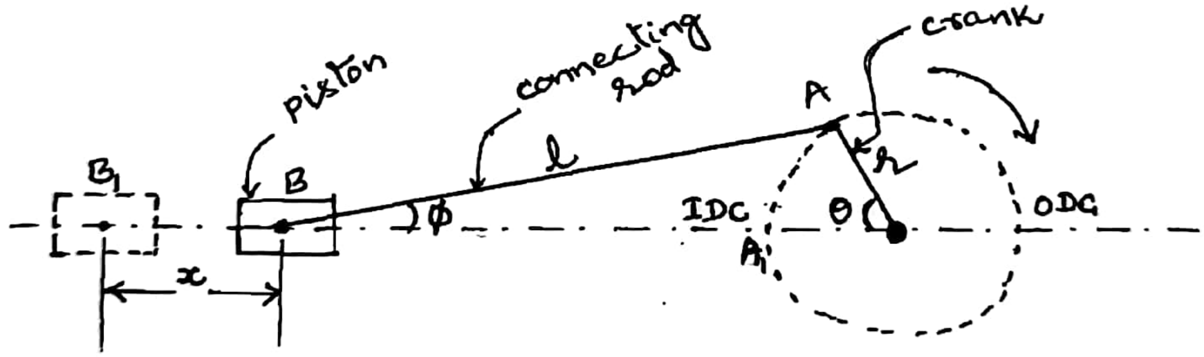
$$= I\omega^2 \frac{C_s}{100}$$ [redacted]

$$\left[\because C_s = \frac{\omega_1-\omega_2}{\omega}\times 100 \qquad \frac{\omega_1-\omega_2}{\omega} = \frac{C_s}{100}\right]$$

$$= \frac{2EC_s}{100}$$

$$\boxed{\Delta E = 0.02\,E\,C_s}$$

$$\left[\because E = \frac{1}{2}I\omega^2 \qquad I\omega^2 = 2E\right]$$



1). Torque on crank shaft $T = F_p \times r\left[\sin\theta + \frac{\sin 2\theta}{2\left[n^2 - \sin^2\theta\right]}\right]$

2). compression ratio of piston $= \frac{V_1}{V_2}$

3). Displacement of the piston $x = r\left[\left[1-\cos\theta\right] + \frac{\sin^2\theta}{2n}\right]$

(or)

$$= r\left[\left[1-\cos\theta\right] + \frac{\left[1-\cos^2\theta\right]}{2n}\right]$$

4). Inertia forces on reciprocating parts

$$F_I = M_R\cdot\omega^2\cdot r\left[\cos\theta + \frac{\cos 2\theta}{n}\right]$$

5). piston effort $F_P = F_L - F_I + W_R$ $(\because W_R = m_R \cdot g)$

6). Net load on piston F_L = pressure × Area

$$= p \times \frac{\pi d^2}{4}$$

7). crank pin effort $F_T = F_P \frac{\sin[\theta+\phi]}{\cos\phi}$

$$\left[\sin\phi = \frac{\sin\theta}{n} \quad \therefore n = \frac{l}{r}\right]$$

8). Turning moment on the crank shaft $T = F_T \times r$

$$T = \left[F_P \frac{\sin[\theta+\phi]}{\cos\phi}\right] r$$

9). Turning moment = Accelerating torque + Resisting torque

Acceleration torque = Turning moment − Resisting torque

where

Acceleration torque $= I \cdot \alpha$

F_P = piston effort

r = Radius of crank $= \frac{\text{stroke}}{2}$

l = length of connecting rod

n = Ratio of the connecting rod length and radius of crank $= \frac{l}{r}$

θ = Angle turned by the crank from inner dead centre

m_R = mass of the reciprocating parts

F_L = Net load on piston = pressure × Area $= p \times A$

W_R = weight of the reciprocating parts $= m_R \cdot g$

Problems

① A horizontal steam engine 20cm diameter by 40cm stroke, connecting rod 100cm makes 160 rpm. The mass of the reciprocating parts is 50kg. When the crank has turned through an angle of 30°, the steam pressure is 4.5 bar.

(a). calculate the turning moment on crank shaft

(b) If the mean resistance torque is 30 N-m and the

Mass of flywheel is 50kg and the radius of gyration 70cm. calculate the acceleration of the flywheel.

Given data:-

$d = 20cm = 0.2m$

$r = \frac{stroke}{2} \Rightarrow \frac{40}{2} \Rightarrow 20cm \Rightarrow 0.2m$

$l = 100cm = 1m$

$N = 160\ rpm$

$\omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 160}{60} \Rightarrow 16.75\ rad/s$

$M_R = 50kg$; $W_R = 50 \times 9.81\ N$ [$\because 1kg = 9.81N$]

$\theta = 30°$

$p = 4.5\ bar \Rightarrow 4.5 \times 10^5 N/m^2$ [$\because 1bar = 10^5 N/m^2$]

$n = \frac{l}{r} \Rightarrow \frac{1}{0.2} \Rightarrow 5$

(i) Turning moment $T = F_p \times \frac{\sin(\theta+\phi)}{\cos\phi} . r$

$F_p = F_L - F_I$

$F_L = p \times A \Rightarrow 4.5 \times 10^5 \times \frac{\pi(0.2)^2}{4} \Rightarrow 14137.1N$

F_I = Inertia force

$= M_R . \omega^2 . r \left[\cos\theta + \frac{\cos 2\theta}{n}\right]$

$= 50 (16.75)^2 . 0.2\left\{\cos 30 + \frac{\cos 2(30)}{5}\right\}$

$= 2710 N$

Effective piston effort $F_p = 14137.1 - 2710$

$= 11427.2N$

$\sin\phi = \frac{\sin\theta}{n}$

$= \frac{\sin 30°}{5} = 0.1$

$\phi = \sin^{-1}(0.1)$

$\phi = 5.74°$

$\therefore T = F_p \frac{\sin(\theta+\phi)}{\cos\phi} \times r \Rightarrow 11427.2 \cdot \frac{\sin(30+5.74)}{\cos 5.74} \times 0.2$

$T = 1341.6\ N\text{-}m$

(ii) Resisting torque = 30N-m (given)

Turning moment = 133.55 N-m

$\therefore$ Acceleration torque = T.m − R.T

$I.\alpha = 1341.6 - 30 \Rightarrow 1311.6N$

$I\alpha = 1311.6$

$\alpha = \frac{1311.6}{I}$

$\alpha = \frac{1311.6}{24.5}$

$\alpha = 53.53\ rad/sec^2$

$I = mK^2$

$= 50 \times 0.7^2$

$= 24.5 kg\text{-}m^2$

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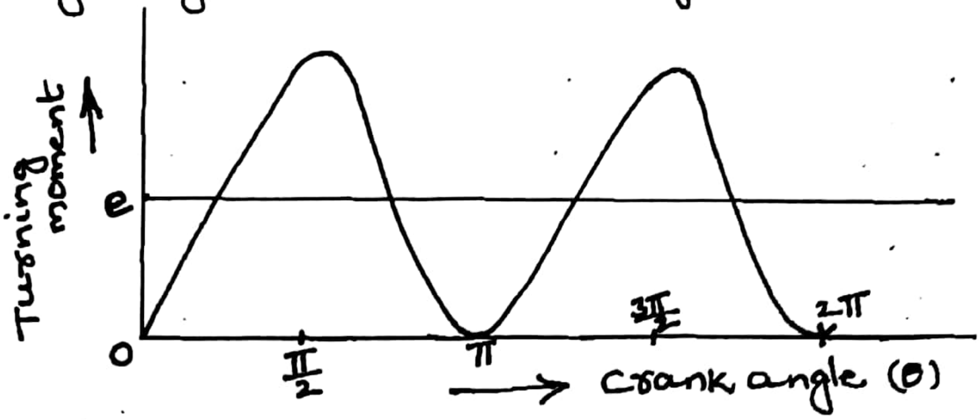
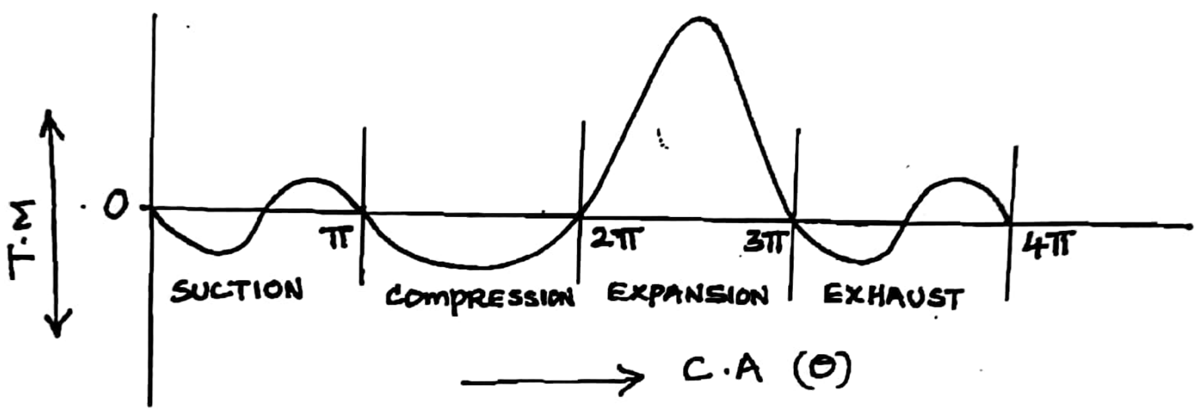
* Turning-moment diagram for :-

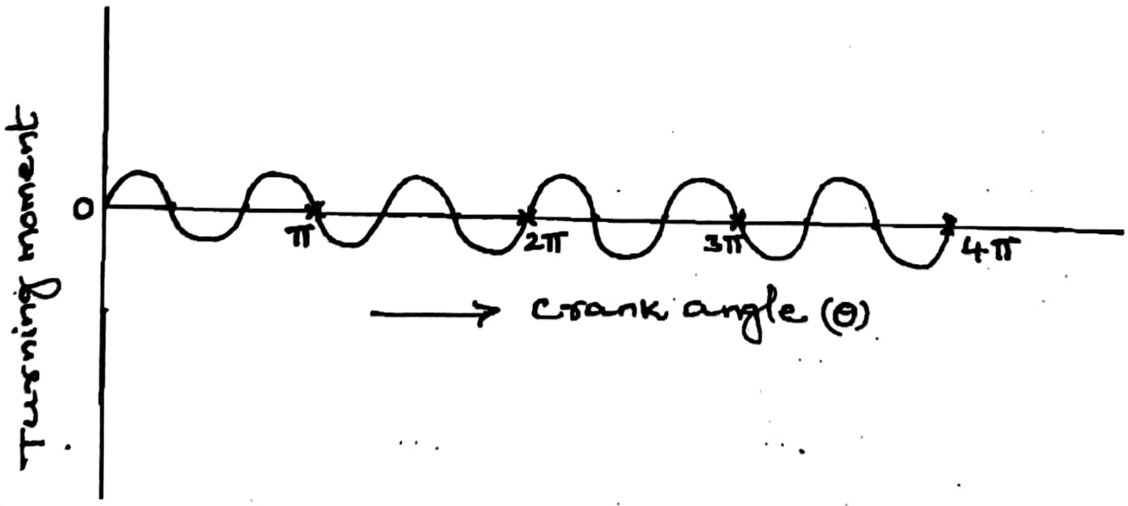
1) Single cylinder double-acting steam engine.



2). Single cylinder four-stroke engine.



3). Multi cylinder engine :-



Mid

Problems:

① The torque delivered by a two-stroke engine is represented by $T = [1000 + 300\sin 2\theta - 500\cos 2\theta]$ N-m. where 'θ' is the angle turned by the crank from the inner-dead centre. The engine speed is 250 rpm. The mass of the flywheel is 400 kg and radius of gyration 400 mm. Determine,

(i) The power developed.

(ii) The total percentage fluctuation of speed,

(iii) The angular acceleration of flywheel, when the crank has rotated through an angle of 60° from the inner-dead centre.

(iv). The maximum angular acceleration and retardation of the flywheel.

Sol The expression for torque being a function of 2θ, the cycle is repeated every 180° of the crank rotation.

$\frac{360}{2} = 180^\circ$ or π rad

$2\theta = \frac{360}{2}$

(i) $$T_{mean} = \frac{1}{\pi}\int_0^{\pi} T \cdot d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi}[1000 + 300\sin 2\theta - 500\cos 2\theta]\,d\theta$$

$$= \frac{1}{\pi}\left[1000\theta + 300\left(-\frac{\cos 2\theta}{2}\right) - 500\left[\frac{\sin 2\theta}{2}\right]\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[1000[\pi] + 150\left(-\cos(2\overset{180^\circ}{\pi})\right) - 250\sin 2(\pi)\right] - [0 - 150 - 0]$$

$T_{mean} = 1000$ N-m

$$\text{power } P = \frac{2\pi N T_{mean}}{60}$$

$$P = \frac{2\pi \times 250 \times 1000}{60} \Rightarrow 26179.9\,W \Rightarrow 26.18\,KW$$

(ii) At any instant $\Delta T = T - T_{mean}$

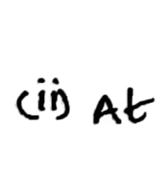
$$= [\cancel{1000} + 300\sin 2\theta - 500\cos 2\theta] - \cancel{1000}$$

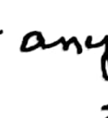
$$\Delta T = 300\sin 2\theta - 500\cos 2\theta$$

ΔT is zero, when $300\sin 2\theta - 500\cos 2\theta = 0$

or $300\sin 2\theta = 500\cos 2\theta$

or $\frac{\sin 2\theta}{\cos 2\theta} = \frac{500}{300} \Longrightarrow \tan 2\theta = \frac{5}{3}$





$2\theta = \tan^{-1}\left(\frac{5}{3}\right)$

$2\theta = 59°$ or $59 + 180° = 239°$

$\theta = 29.5$ or $119.5°$.

Maximum fluctuation of energy $[e_{max}] = \int_{29.5}^{119.5^\circ} \Delta T . d\theta$

$$= \int_{29.5^\circ}^{119.5^\circ} [300 \sin 2\theta - 500 \cos 2\theta] \, d\theta$$

$$= \left[-300 \frac{\cos 2\theta}{2} - 500 \frac{\sin 2\theta}{2}\right]_{29.5}^{119.5}$$

$e_{max} = 583.1$ N-m

co-efficient of fluctuation of speed $K = \frac{e}{I\omega^2}$ $[\because I = mk^2]$

$$= \frac{e}{mk^2 . \omega^2}$$

$k = 400$mm
$= 0.4$m
$N = 250$ rpm

$$= \frac{583.1}{400(0.4)^2 \times \left[\frac{2\pi(250)}{60}\right]^2} \qquad \because \omega = \frac{2\pi N}{60}$$

$K = 0.01329$
$= 1.329\%$

(iii). The angular acceleration of flywheel, when the crank has rotated through an angle of 60° from the inner-dead centre.

$\Delta T = 300 \sin 2\theta - 500 \cos 2\theta$

when $\theta = 60^\circ$

$\Delta T = 300 \sin 2(60^\circ) - 500 \cos 2(60^\circ)$
$= 509.8$ N-m

$\Delta T = I . \alpha \implies mk^2 . \alpha$

$509.8 = 400(0.4)^2 . \alpha$

$\alpha = 7.966$ rad/sec^2.

(iv). The maximum acceleration and retardation of the flywheel.

$$\frac{d}{d\theta}[\Delta T] = \frac{d}{d\theta}\left[300 \sin 2\theta - 500 \cos 2\theta\right] d\theta = 0$$

$300 \cos 2\theta (2) - 500(\sin 2\theta)(2) = 0$

$600 \cos 2\theta + 1000 \sin 2\theta = 0$

$1000 \sin 2\theta = -600 \cos 2\theta$

$\frac{\sin 2\theta}{\cos 2\theta} = \frac{-600}{1000}$

$\tan 2\theta = \frac{-6}{10}$

$2\theta = \tan^{-1}\left[\frac{-6}{10}\right]$

-30.96
$+180.00$
149.04

-ve so Next angle we can take

$2\theta = -30.96^\circ$ Next angle

$2\theta = 149.04^\circ$ and 329.04°

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when $2\theta = 149.04^\circ$

$\Delta T = 300 \sin 2\theta - 500 \cos 2\theta \Rightarrow 300 \sin[149.04^\circ] - 500 \cos[149.04^\circ]$

$\Delta T = 583.1$ N-m

when $2\theta = 329.04^\circ$

$\Delta T = 300 \sin(329.04^\circ) - 500 \cos(329.04^\circ)$

$= -583.1$ N-m

As values of ΔT at maximum and minimum torque are same, maximum acceleration is equal to maximum retardation.

$$\Delta T = I \cdot \alpha$$

$$= (mK^2)\,\alpha$$

$$583.1 = 400\,(0.4)^2 \cdot \alpha$$

$$\alpha = 9.11 \text{ rad/sec}^2$$

Maximum acceleration or retardation $\alpha = 9.11$ rad/sec

2011 ✓

(2) A machine is coupled to a two-stroke engine which produces a torque of $[800 + 180 \sin 3\theta]$ N-m, where θ is the crank angle. The mean engine speed is 400 rpm. The flywheel and the other rotating parts attached to the engine have a mass of 350 kg at a radius of gyration of 220 mm. Calculate

(i) The power of the engine

(ii) The total fluctuation of speed of the flywheel when

(a) The resisting torque is constant.

(b) The resisting torque is $[800 + 80 \sin\theta]$ N-m.

Sol

$m = 350$ kg

$N = 400$ rpm $\Rightarrow \omega = \frac{2\pi N}{60} \Rightarrow \frac{2\pi \times 400}{60} \Rightarrow 41.89$ rad/sec

$k = 220$ mm

The expression for torque being a function of 3θ, the cycle is repeated after every 120° $\left(\frac{2\pi}{3}\right)$ of the crank rotation.

$\left\{\frac{360}{3} = 120^\circ \text{ or } \frac{2\pi}{3}\right\}$

(1) $T_{mean} = \frac{1}{\frac{2\pi}{3}} \int_0^{\frac{2\pi}{3}} T \cdot d\theta$

$$= \frac{3}{2\pi} \int_0^{\frac{2\pi}{3}} [800 + 180 \sin 3\theta]\, d\theta$$

$$= \frac{3}{2\pi} \left[800\theta - 180 \frac{\cos 3\theta}{3}\right]_0^{\frac{2\pi}{3}} \Rightarrow 800 \text{ N-m}$$

power $P = \frac{2\pi N T}{60} \Longrightarrow \frac{2\pi \times 400 \times 800}{60}$

$P = 33512\ W \Longrightarrow 33.512\ kW$

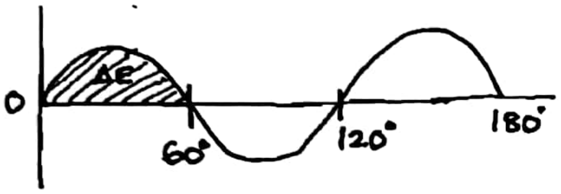
(ii) (a) At any instant, $\Delta T = T - T_{mean}$

$= \cancel{800} + 180 \sin 3\theta - \cancel{800}$

$\Delta T = 180 \sin 3\theta$

ΔT is zero, when $180 \sin 3\theta = 0$

$\sin 3\theta = 0$



$3\theta = \sin^{-1}(0)$

$3\theta = 0$

$3\theta = 0$ or $180°$

$\theta = 0$ or $60°$

e_{max} = max fluctuation of energy

$= \int_0^{60} \Delta T \cdot d\theta$

$= \int_0^{60} (180 \sin 3\theta) d\theta \Longrightarrow \left[180 \left(-\frac{\cos 3\theta}{3}\right)\right]_0^{60}$

$= \frac{-180 \cos 3(60)}{3} - \left[-\frac{180 \cos 3(0)}{3}\right]$

$= 120\ N\text{-}m$

Co-efficient of fluctuation of speed $K = \frac{e}{I\omega^2}$ $(\because I = mk^2)$

$K = \frac{e}{mk^2 \cdot \omega^2} \Longrightarrow \frac{120}{350(0.22)^2(41.89)^2}$

$K = 0.00404 \Longrightarrow 0.404\%$

(b) ΔT = T of engine − T of machine

$= \cancel{800} + 180 \sin 3\theta - [\cancel{800} + 80 \sin \theta]$

$\Delta T = 180 \sin 3\theta - 80 \sin \theta$

ΔT is zero when $180 \sin 3\theta - 80 \sin \theta = 0$

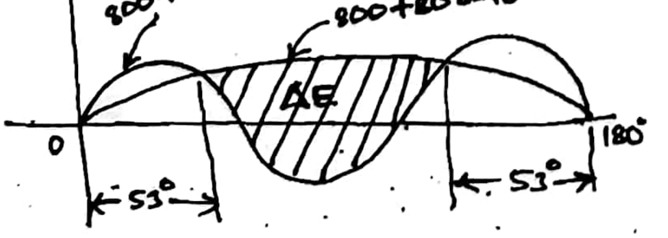
$180 \sin 3\theta = 80 \sin \theta$

$\boxed{\because \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta}$

$180 [3 \sin \theta - 4 \sin^3 \theta] = 80 \sin \theta$

$180 \sin \theta [3 - 4 \sin^2 \theta] = 80 \sin \theta$

$3 - 4 \sin^2 \theta = \frac{80 \cancel{\sin \theta}}{180 \cancel{\sin \theta}}$



$\sin^2 \theta = 3 - \frac{8}{18}$

$\sin^2 \theta = 0.639$

$\sin \theta = \pm 0.799$

$\theta = \pm 53°$ and $\pm 127°$

180 − 53 = 127

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$$e_{max} = \int_{53^\circ}^{127^\circ} \Delta T \, d\theta \Longrightarrow \int_{53^\circ}^{127} [180 \sin 3\theta - 80 \sin\theta] \, d\theta$$

$$= \left[\frac{180(-\cos 3\theta)}{3} + 80\cos\theta \right]_{53^\circ}^{127^\circ}$$

$$= -60\cos 3(127^\circ) + 80\cos 127^\circ - [-60\cos 159^\circ + 80\cos 53^\circ]$$

$3(53^\circ)$

$$= -208.7 \text{ N-m}$$

$$K = \frac{e}{I\omega^2} \Longrightarrow \frac{e}{mk^2 \cdot \omega^2} \Longrightarrow \frac{208.7}{350 \times (0.22)^2 \times [41.89]^2}$$

$$K = 0.00703$$

$$K = 0.703\%$$

Mid

③ The turning moment diagram for a multi cylinder engine has been drawn to a vertical scale of 1 mm = 650 N-m and a horizontal scale of 1 mm = 4.5°. The areas above and below the mean torque line are −28, +380, −260, +310, −300, +242, −380, +265 and −229 mm².

The fluctuation of speed is limited to ± 1.8% of the mean speed which is 400 rpm. Density of the rim material is 7000 kg/m³ and width of the rim is 4.5 times its thickness. The centrifugal stress [hoop stress] in the rim material is limited to 6 N/mm². Neglecting the effect of the boss and arms, determine the diameter and cross-section of the flywheel rim.

Sol

$\rho = 7000 \text{ kg/m}^3$; $\sigma = 6 \text{ N/mm}^2$ $\left[\because 1\text{mm}^2 = \frac{1}{10^6}\text{m}^2\right]$

$N = 400$ rpm $\qquad = 6 \times 10^6 \text{ N/m}^2$

$K = \pm 1.8\%$

$b = 4.5t \qquad = 0.018 \qquad \left\{\frac{1.8}{100} = 0.018\right\}$

$$\sigma = \rho V^2$$

$$6 \times 10^6 = 7000 V^2$$

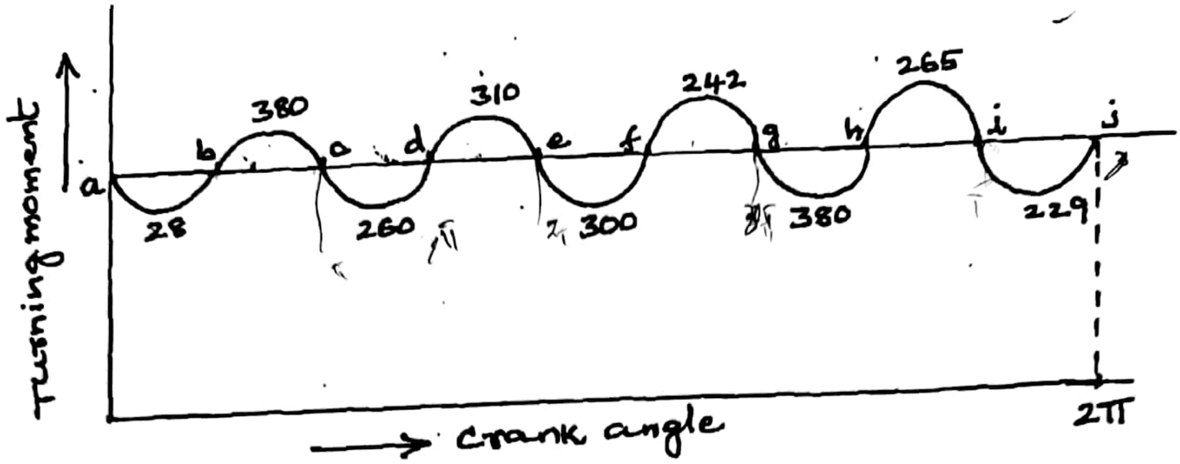
$$V = 29.28 \text{ m/sec}$$

$$V = \frac{\pi D N}{60}$$

$$29.28 = \frac{\pi D \times 400}{60}$$

$$D = 1.398 \text{ m}$$

Radius of gyration $k = \frac{D}{2} \Longrightarrow \frac{1.398}{2} \Rightarrow 0.699 \text{ m}$



Let

flywheel kinetic energy at $a = E$

at $b = E - 28$

at $c = E - 28 + 380 = E + 352$

at $d = E + 352 - 260 = E + 92$

at $e = E + 92 + 310 = E + 402$ ✓

at $f = E + 402 - 300 = E + 102$

at $g = E + 102 + 242 = E + 344$

at $h = E + 344 - 380 = E - 36$ ✓

at $i = E - 36 + 265 = E + 229$

at $j = E + \cancel{229} - \cancel{229} = E$

Maximum energy $= E + 402$ ——— [at point e]

Minimum energy $= E - 36$ ——— [at point h]

Maximum fluctuation of energy.

$e_{max} = [\cancel{E} + 402] - [\cancel{E} - 36] \times$ Horizontal scale $\times$ Vertical scale

$= 438 \times \left(4.5 \times \frac{\pi}{180}\right) \times 650$

$= 22360$ N-m

$= 438\ mm^2$
$= 438\ mm \times mm$
$= 438 \times \left(4.5 \times \frac{\pi}{180}\right) \times 650$

K = coefficient of fluctuation of speed

$K = \frac{e_{max}}{I\omega^2} \Longrightarrow \frac{e_{max}}{mk^2 \cdot \omega^2}$

$0.018 = \frac{22360}{m[0.699]^2 \left[\frac{2\pi \times 400}{60}\right]^2} \Longrightarrow m = 1449\ kg$

$\therefore$ density $= \frac{\text{Mass}}{\text{Volume}}$

Mass of the flywheel $m = \rho \cdot V$

volume $= \pi d \times b \times t$
$= \pi d \times 4.5t \times t$

$1449 = 7000 [\pi d \times 4.5t \times t]$

$t = 0.1023\ m \Longrightarrow 102.3\ mm$

$b = 4.5(t) \Longrightarrow 4.5(102.3) \Longrightarrow 460.5\ mm$

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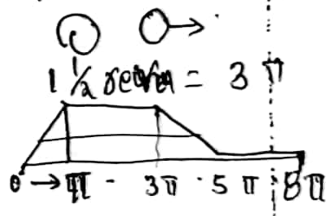
④ In a machine, the intermittent operations demand the torque to be applied as follows.
- During the first half revolution, the torque increases uniformly from 800 N-m to 3000 N-m.
- During next one revolution, the torque remains constant
- During next one revolution, the torque decreases uniformly from 3000 N-m to 800 N-m.
- During last $1\frac{1}{2}$ revolution, the torque remains constant

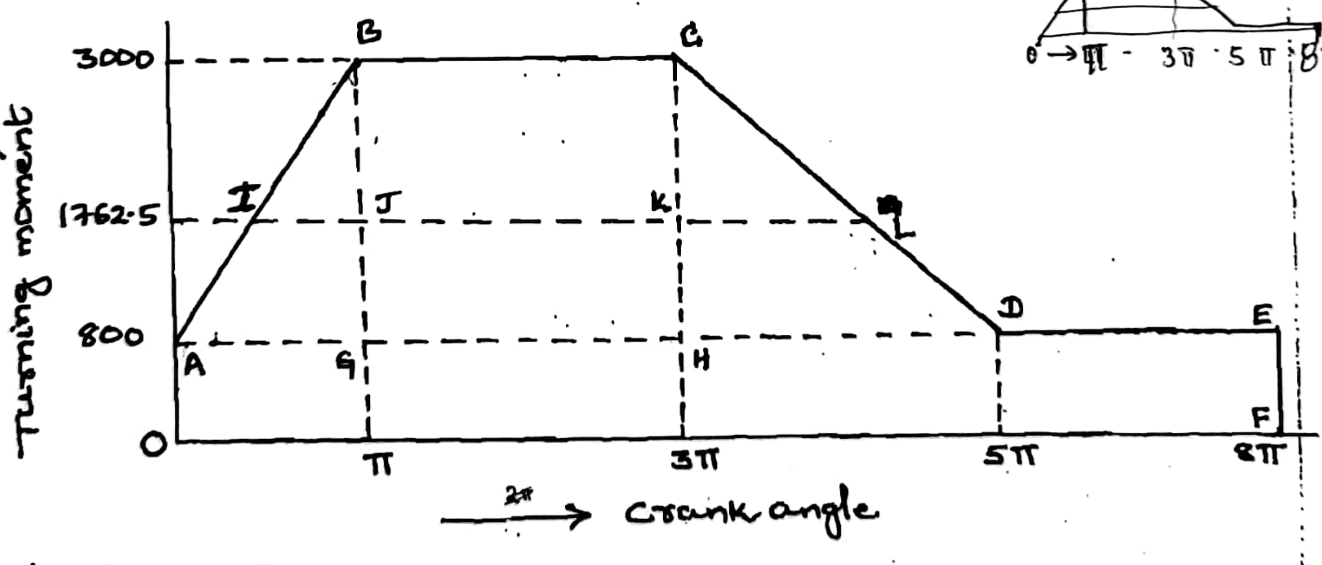
Thus a cycle is completed in 4 revolutions. The motor to which the machine is coupled exerts a constant torque at a mean speed of 250 rpm. A flywheel of mass 1800 kg and radius of gyration of 500 mm is fitted to the shaft. Determine (i) The power of the motor (ii) The total fluctuation of speed of the machine shaft.

1 revolution $= 2\pi$
half revolution $= \pi$
$1\frac{1}{2}$ revolution $= 3\pi$



Sol

$m = 1800$ kg
$K = 500$ mm $= 0.5$ m
$N = 250$ rpm.



(i) from figure

Torque for one complete cycle T = Area of OABCDEF

T = Area OAEF + Area ABG + Area BGCH + Area CHD

$= 8\pi \times 800 + \frac{1}{2}\pi(3000-800) + 2\pi(3000-800) + \frac{1}{2}[2\pi](3000-800)$

$= 44296$ N-M

$T_{mean} = \frac{44296}{8\pi} \longrightarrow 1762.5$ N-M

$$\text{power } p = \frac{2\pi N T_m}{60} \Longrightarrow \frac{2\pi \times 250 \times 1762.5}{60}$$

$$p = 46142 \text{ watts}$$

$$p = 46.142 \text{ KW}$$

(ii) from ΔABG

$$\frac{IJ}{AG} = \frac{BJ}{BG}$$

$$IJ = \frac{BJ \times AG}{BG}$$

$$= \frac{(3000 - 1762.5)\pi}{(3000 - 800)} \Longrightarrow iJ = 1.767$$

from ΔCHD

$$\frac{KL}{HD} = \frac{CK}{CH}$$

$$KL = \frac{CK \times HD}{CH}$$

$$= \frac{(3000 - 1762.5)\,2\pi}{(3000 - 800)} \Longrightarrow 3.534$$

Fluctuation of energy is equal to the Area above the Mean torque line.

$$e = \text{Area of } IBCL$$

$$= \text{Area } IBJ + \text{Area } BJCK + \text{Area } CKL$$

$$= \left(\frac{1}{2}(1.769) \times (3000 - 1762.5)\right) + 2\pi[3000 - 1762.5] + \frac{1}{2}[3.534][3000 - 1762.5]$$

$$e = 11055 \text{ N-m}$$

Total fluctuation of speed $k = \dfrac{e}{I\omega^2}$

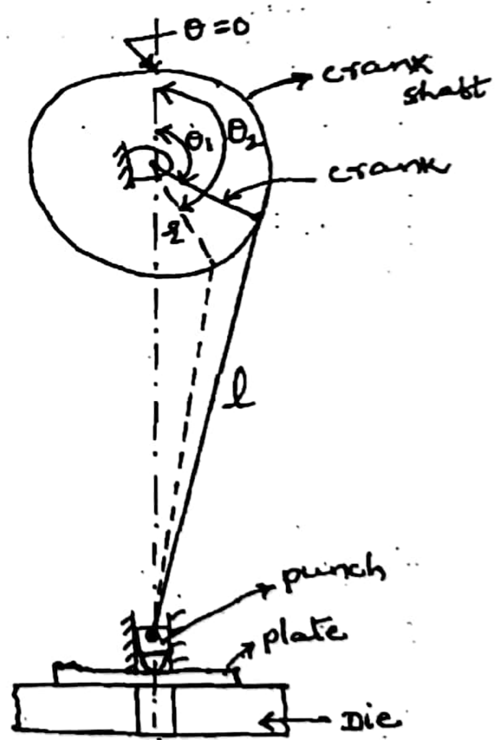
$$= \frac{e}{mk^2 \cdot \omega^2} \qquad [\because I = mk^2]$$

$$k = \frac{11055}{1800\,(0.5)^2 \times \left[\frac{2\pi \times 250}{60}\right]^2} \qquad \left[\because \omega = \frac{2\pi N}{60}\right]$$

$$k = 0.0358$$

$$k = 3.58\%$$

Flywheel in punching press :- A punching press is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism from figure, we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied. on the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from $\theta = \theta_1$ to $\theta = \theta_2$ due to much load than the energy supplied.



Let

E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched

d_1 = diameter of the hole punched

t_1 = thickness of the plate

τ_u = ultimate shear stress for the plate material.

Maximum shear force required for punching

$$F_s = \text{Area sheared} \times \text{ultimate shear stress}$$
$$= \pi d_1 t_1 \times \tau_u$$

∴ work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

$\therefore E = \frac{1}{2} mgh = \frac{1}{2} w \cdot h$

Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1.

Turning Moment and Flywheel

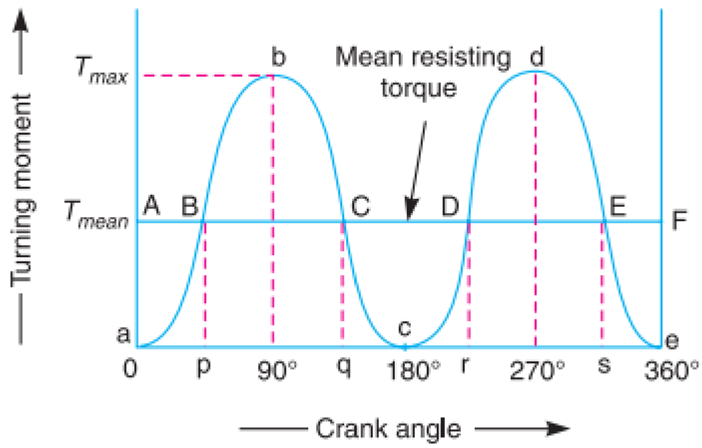
The turning moment diagram (also known as crank- effort diagram) is the graphical representation of the turning moment or crank-effort for various positions of the crank. It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa.

Turning diagram for a single Cylinder Double acting Steam Engine:

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. 16.1. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

the turning moment on the crankshaft,

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$



Turning moment diagram for a single cylinder, double acting steam engine.

FP = Piston effort, r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.This is shown by the curve abc in Fig. 16.1 and it represents the turning moment diagram for outstroke. The curve cde is the turning moment diagram for instroke and is somewhat similar to the curve abc.

Notes: 1. When the turning moment is positive (i.e. when the engine torque is more than the mean resisting torque) as shown between points B and C (or D and E) in Fig. 16.1, the crankshaft accelerates and the work is done by the steam.

When the turning moment is negative (i.e. when the engine torque is less than the mean resisting torque) as shown between points C and D in Fig. 16.1, the crankshaft retards and the work is done on the steam

. T = Torque on the crankshaft at any instant, and

Tmean=Mean resisting torque.

Then accelerating torque on the rotating parts of the engine

= T – Tmean

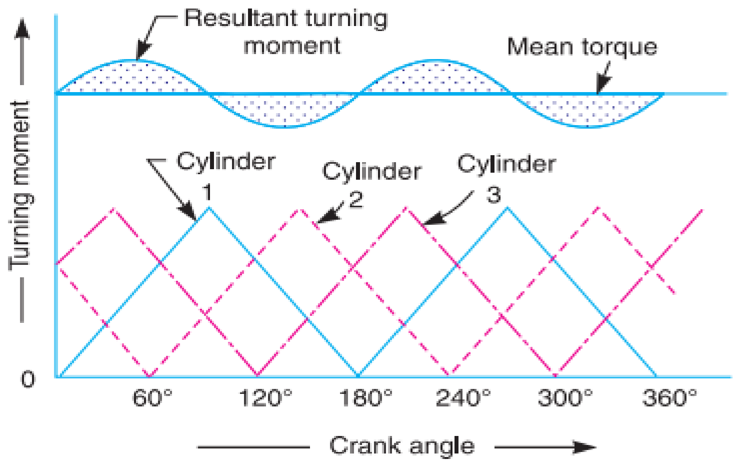
If (T –Tmean) is positive, the flywheel accelerates and if (T – Tmean) is negative, then the flywheel retards.

Turning Moment Diagram for a Four Stroke Cycle Internal Combustion Engine:

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. 16.2. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4 π radians).

Turning moment diagram for a four stroke cycle internal combustion engine..

Turning Moment Diagram for a Multi-cylinder Engine:



Turning moment diagram for a multi-cylinder engine.

Fluctuation of Energy:

Let the energy in the flywheel at A = E, then from Fig.

Let the energy in the flywheel at A = E, then from Fig. 16.4, we have

Energy at B = E + a1

Energy at C = E + a1– a2

Energy at D = E + a1 – a2 + a3

Energy at E = E + a1 – a2 + a3 – a4

Energy at F = E + a1 – a2 + a3 – a4 + a5

Energy at G = E + a1 – a2 + a3 – a4 + a5 – a6 = Energy at A (i.e. cycle repeats after G)

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in flywheel

= E + a

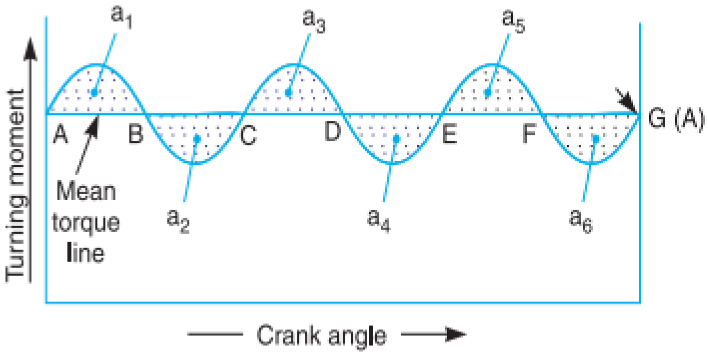
Minimum energy in the flywheel

= E + a1 – a2 + a3 – a4

∴ Maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy

=(E + a1) – (E + a1 – a2 + a3 – a4) = a2 – a3 + a4



Determination of maximum fluctuation of energy.

Coefficient of Fluctuation of Energy:

It may be defined as the ratio of the maximum fluctuation of energy to the work done per cycle. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

Work done per cycle = Tmean × θ

T mean = Mean torque, and

θ = Angle turned (in radians), in one revolution.

=2π, in case of steam engine and two stroke internal combustion engines

=4π, in case of four stroke internal combustion engines.

The mean torque (Tmean) in N-m may be obtained by using the following relation

$T_{mean} = \frac{P*60}{2\pi N}$

Coefficient of Fluctuation of Speed:

The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed.

N1 and N2 = Maximum and minimum speeds in r.p.m. during the cycle, and

N = Mean speed in r.p.m $\frac{N1+N2}{2}$

Coefficient of fluctuation of speed

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

Note. The reciprocal of the coefficient of fluctuation of speed is known as coefficient of steadiness and is denoted by m

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

Flywheel

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

- In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke.
- For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines.
- The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.
- A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.
- In machines where the operation is intermittent like punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

Energy stored in Flywheel:

When a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let m = Mass of the flywheel in kg,

k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m2 = m.k2,

N 1 and N2 = Maximum and minimum speeds during the cycle in r.p.m., ω1 and ω2 = Maximum and minimum angular speeds during the cycle in rad/s,

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

$$\omega = \text{ Mean angular speed during the cycle in rad/s } = \frac{\omega_1 + \omega_2}{2}$$

$$C_S = \text{Coefficient of fluctuation of speed} = \frac{N_1 - N_2}{N} \text{or} \frac{\omega_1 + \omega_2}{\omega}$$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I.\omega^2 = \frac{1}{2} m.k^2.\omega^2$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

ΔE = Maximum K.E. – Minimum K.E.

$$= \frac{1}{2} I.(\omega_1)^2 - \frac{1}{2} I.(\omega_2)^2 = \frac{1}{2} I\left[(\omega_1)^2 - (\omega_2)^2\right]$$

$$= \frac{1}{2} I(\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \qquad \text{..................(i)}$$

$$= I.\omega^2\left(\frac{\omega_1 - \omega_2}{\omega}\right)$$

$$= I.\omega^2 C_S = m.k^2.\omega^2 C_S \qquad \text{....................(ii)}$$

$$= 2.E.C_S \qquad \text{................(iii)}$$

he

radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting k = R, in equation (ii), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

v = Mean linear velocity (*i.e.* at the mean radius) in m/s

Problems:

The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds.

Given : m = 6.5 t = 6500 kg ; k = 1.8 m ; Δ E = 56 kN-m = 56 × 103 N-m ; N = 120 r.p.m.

Let N1 and N2 = Maximum and minimum speeds respectively. We know that fluctuation of energy (Δ E),

$$56 \times 10^3 = \frac{\pi^2}{900} \times m.k^2 . N (N_1 - N_2) = \frac{\pi^2}{900} \times 6500 (1.8)^2\ 120 (N_1 - N_2)$$

$$= 27\,715 (N_1 - N_2)$$

$\therefore \quad N_1 - N_2 = 56 \times 10^3 / 27\,715 = 2$ r.p.m. ...(*i*)

We also know that mean speed (N),

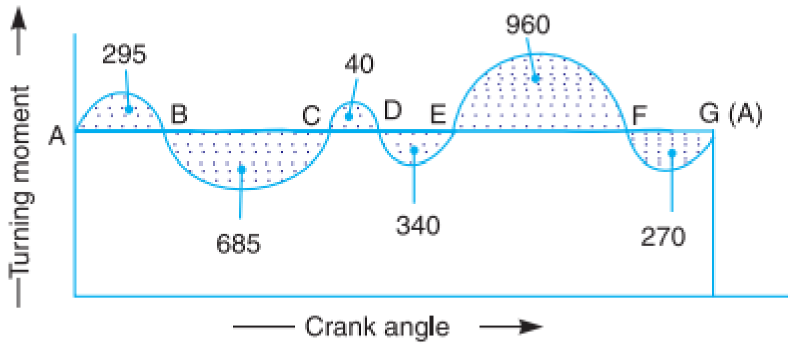
$$120 = \frac{N_1 + N_2}{2} \text{ or } N_1 + N_2 = 120 \times 2 = 240 \text{ r.p.m.} \quad ...(ii)$$

From equations (*i*) and (*ii*),

$N_1 = 121$ r.p.m., and $N_2 = 119$ r.p.m. **Ans.**

2. The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 5 N-m ; crank angle, 1 mm = 1°. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line taken in order are 295, 685, 40, 340, 960, 270 mm2. The rotating parts are equivalent to a mass of 36 kg at a radius of gyration of 150 mm. Determine the coefficient of fluctuation of speed when the engine runs at 1800 r. p . m.

Given : m = 36 kg ; k = 150 mm = 0.15 m ; N = 1800 r.p.m. or ω = 2 π × 1800/60 = 188.52 rad /s



Since the turning moment scale is 1 mm = 5 N-m and crank angle scale is 1 mm = 1° = π /180 rad, therefore,

1 mm2 on turning moment diagram

$$= 5 \times \frac{\pi}{180} = \frac{\pi}{36} \text{ N-m}$$

Let the total energy at A = E,

Energy at B = E + 295

... (Maximum energy)

Energy at C = E + 295 – 685 = E – 390

Energy at D = E – 390 + 40 = E – 350

Flywheel of an electric motor.

Energy at E = E – 350 – 340 = E – 690 ...(Minimum energy)

Energy at F = E – 690 + 960 = E + 270

Energy at G = E + 270 – 270 = E = Energy at A

We know that maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy =(E + 295) – (E – 690) = 985 mm^2

$$= 985 \times \frac{\pi}{36} = 86 \text{ N-m} = 86 \text{ J}$$

Let C_S = Coefficient of fluctuation of speed.

We know that maximum fluctuation of energy (ΔE),

$$86 = m.k^2\,\omega^2.C_S = 36 \times (0.15)^2 \times (188.52)^2\, C_S = 28\,787\, C_S$$

$$\therefore \quad C_S = 86 / 28\,787 = 0.003 \text{ or } 0.3\% \quad \textbf{Ans.}$$

Dimensions of the Flywheel Rim

Consider a rim of the flywheel as shown in Fig

D = Mean diameter of rim in metres,

R = Mean radius of rim in metres, A = Cross-sectional area of rim in m2, ρ=Density of rim material in kg/m3, N = Speed of the flywheel in r.p.m.,

ω = Angular velocity of the flywheel in rad/s, v = Linear velocity at the mean radius in m/s

= ω .R = π D.N/60, and

σ = Tensile stress or hoop stress in N/m2 due to the centrifugal force

Consider a small element of the rim as shown shaded in Fig. 16.17. Let it subtends an angle δθ at the centre of the flywheel.

Volume of the small element = A × R.δθ

∴ Mass of the small element

dm = Density × volume = ρ.A.R.δθ

nd centrifugal force on the element, acting radially outwards,

$dF = dm.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$

Vertical component of dF= dF.sin θ = ρ.A.R2.ω2.δθ.sin θ

Total vertical upward force tending to burst the rim across the diameter X Y.

$$= \rho.A.R^2.\omega^2 \int_0^{\pi} \sin\theta.d\theta = \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^{\pi}$$

$$= 2\rho.A.R^2.\omega^2 \qquad \ldots (i)$$

This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by 2P, such that

$$2P = 2\sigma.A \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

$$\sigma = \rho.R^2.\omega^2 = \rho.v^2 \quad \ldots(\because v = \omega.R)$$

$$\therefore \quad v = \sqrt{\frac{\sigma}{\rho}} \quad \ldots(iii)$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi\, D.A.\rho$$

$$\therefore \quad A = \frac{m}{\pi.D.\rho} \quad \ldots(iv)$$

From equations (iii) and (iv), we may find the value of the mean radius and cross-sectional area of the rim.

Note: If the cross-section of the rim is a rectangular, then

A = b × t

where

b = Width of the rim, and t = Thickness of the rim.

Problem: The turning moment diagram for a multi-cylinder engine has been drawn to a scale of 1 mm to 500 N-m torque and 1 mm to 6° of crank displacement. The intercepted areas between output torque curve and mean resistance line taken in order from one end, in sq. mm are

– 30, + 410, – 280, + 320, – 330, + 250, – 360, + 280, – 260 sq. mm, when the engine is running at 800 r.p.m.

The engine has a stroke of 300 mm and the fluctuation of speed is not to exceed ± 2% of the mean speed. Determine a suitable diameter and cross-section of the flywheel rim for a limiting value of the safe centrifugal stress of 7 MPa. The material density may be assumed as 7200 kg/m3. The width of the rim is to be 5 times the thickness.

Given : N = 800 r.p.m. or ω = 2π × 800 / 60 = 83.8 rad/s; *Stroke = 300 mm ; σ = 7 MPa = 7 × 106 N/m^2 ; ρ = 7200 kg/m^3

Since the fluctuation of speed is ± 2% of mean speed, therefore total fluctuation of speed,

ω1 – ω2 = 4% ω = 0.04 ω and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

Diameter of the flywheel rim

D = Diameter of the flywheel rim in metres, and v = Peripheral velocity of the flywheel rim in m/s.

We know that centrifugal stress (σ),

7 × 106 = ρ.v^2 = 7200 v2 or v^2 = 7 × 106/7200 = 972.2

v = 31.2 m/s

∴ We know that $v = \pi D.N/60$

$D = v \times 60 / \pi N = 31.2 \times 60/\pi \times 800 = 0.745$ m

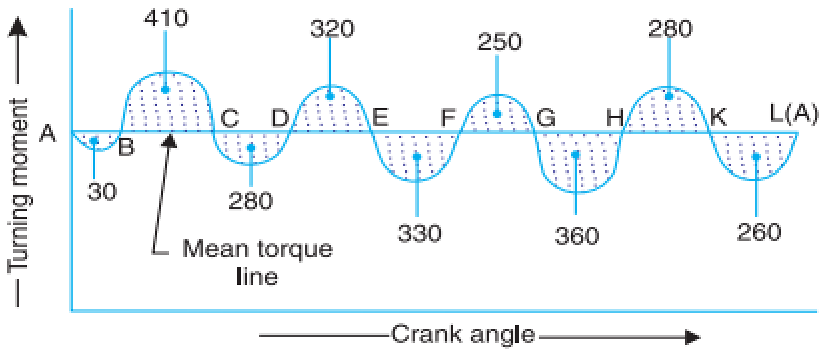
Cross-section of the flywheel rim

t = Thickness of the flywheel rim in metres, and b = Width of the flywheel rim in metres = 5 t

∴ Cross-sectional area of flywheel rim,

$A = b.t = 5 t \times t = 5 t^2$

First of all, let us find the mass (m) of the flywheel rim. The turning moment diagram is



Since the turning moment scale is 1 mm = 500 N-m and crank angle scale is 1 mm = 6° = π /30 rad, therefore 1 mm2 on the turning moment diagram

= 500 × π / 30 = 52.37 N-m

Let the energy at A = E, then referring to Fig

Energy at B = E – 30 . . . (Minimum energy)

Energy at C = E – 30 + 410 = E + 380

Energy at D = E + 380 – 280 = E + 100

Energy at E = E + 100 + 320 = E + 420 . . . (Maximum energy)

Energy at F = E + 420 – 330 = E + 90

Energy at G = E + 90 + 250 = E + 340

Energy at H = E + 340 – 360 = E – 20

Energy at K = E – 20 + 280 = E + 260

Energy at L = E + 260 – 260 = E = Energy at A

We know that maximum fluctuation of energy,

ΔE = Maximum energy – Minimum energy

$=(E + 420) – (E – 30) = 450 \text{ mm}^2 = 450 \times 52.37 = 23\,566$ N-m

We also know that maximum fluctuation of energy (ΔE),

$23\,566 = m.v^2.C_S = m \times (31.2)^2 \times 0.04 = 39$ m

m = 23566 / 39 = 604 kg

∴ We know that mass of the flywheel rim (m),

$604 = \text{Volume} \times \text{density} = \pi D.A.\rho$

$= \pi \times 0.745 \times 5t^2 \times 7200 = 84\,268\ t^2$

$t^2 = 604 / 84\,268 = 0.007\,17\ m^2$ or $t = 0.085\ m = 85\ mm$ Ans. $b = 5t = 5 \times 85 = 425\ mm$